Modification of the quasi-levels of an electron in a laser field due to radiative corrections

1976 J. Phys. A: Math. Gen. 9 2171

(http://iopscience.iop.org/0305-4470/9/12/021)

J. Phys. A: Math. Gen., Vol. 9, No. 12, 1976. Printed in Great Britain. © 1976

Modification of the quasi-levels of an electron in a laser field due to radiative corrections

W Becker† and H Mitter‡

† Institut für Theoretische Physik der Universität Tübingen, D-7400 Tübingen, Germany
‡ Institut für Theoretische Physik der Universität Graz, A-8010 Graz, Austria

Received 25 May 1976, in final form 26 July 1976

Abstract. The mass operator of an electron in a plane wave field of arbitrary polarization is calculated to lowest order of α (but exactly with respect to the external field). It is represented by a threefold integral over elementary functions or a double integral over exponential integral functions. For a monochromatic plane wave and circular polarization the number of integrations reduces by one and the mass operator turns out to be almost diagonal. In this case the corrected propagator is obtained and the modifications of the quasi-levels of the electron are investigated. The property of the mass operator and the propagator to be almost diagonal is related to a special symmetry of the circularly polarized field and found to be valid to all orders.

1. Introduction

Due to constant progress in the development of high power lasers the investigation of the behaviour of electrons in strong electromagnetic fields has found growing interest. Since the laser frequency is small compared to characteristic frequencies in high-energy effects, the 'zero frequency' approximation of the laser field by a constant, crossed field has been used frequently in this context (Ritus 1970, 1972). If, however, the values of the relevant parameters are large enough to yield experimentally observable effects, this approximation can become inadequate. Electrons in a field periodic in time or space–time exhibit a quasi-level structure (Nikishov and Ritus 1964, Reiss and Eberly 1966, Ritus 1967, Zeldovich 1967). Essentials of all processes involving electrons in laser fields (stimulated bremsstrahlung, Compton scattering, Møller scattering etc) are easily understood in terms of transitions between these quasi-levels (cf Mitter 1975, Becker 1976). To lowest order the dispersion law of the quasi-levels in a monochromatic laser field is simply

$$(p \pm nk)^2 = \kappa_*^2$$

where k denotes the wavevector of the laser field, κ_* is a (real) effective mass and the integer n enumerates the different levels. This dispersion law is modified by radiative corrections of order α, α^2 etc, which influence all processes in this way. The corrections are of particular importance, since the unmodified dispersion law causes resonance infinities in the cross sections for some processes, e.g. stimulated Compton and Møller scattering (Oleinik 1967a, b). These infinities become finite resonances, if the modified dispersion law is used, in which the quasi-levels get a finite width of order α.

In order to obtain the modified levels, the corrected propagator must be calculated, which in turn involves the mass operator. In the second section we calculate to lowest order in α the Volkov transform of this operator, which is the simplest and most adequate representation of all quantities referring to electrons in laser fields, especially with respect to renormalization problems. Since the calculational techniques have been discussed at length elsewhere (Mitter 1975) we give no details of the evaluation. The result can be reduced to a triple integral of elementary functions or a double integral of exponential integrals. For a monochromatic and circularly polarized plane wave field one of these integrations can be performed exactly and we obtain a remarkable structure: the Volkov transform of the mass operator contains only terms of order $\delta(p-p')$ and $\delta(p-p'\pm k)$. This structure is the consequence of a higher symmetry of the circularly polarized field and is shown to hold to all orders in α in appendix 2. A similar phenomenon occurs for the vacuum polarization tensor (Becker and Mitter 1975). The 'almost diagonal' structure of the mass operator allows for a comparatively easy calculation of the propagator, which exhibits the same structure (§ 3). The poles of the propagator are discussed extensively in an approximative way by means of numerical estimates. A more general discussion and explicit formulae are given in appendix 1. Our approximate results agree with those obtained by other authors (Baier *et al* 1975) from averaging the mass operator on the mass shell. It turns out, however, that the poles of the propagator are not correctly obtained by this method for very small energies ($2(pk)/\kappa^2 \ll \alpha$), so that our general formulae have to be used in this case. The modified quasi-levels are obtained in § 4 by a transformation from the Volkov to the ordinary momentum representation of the propagator.

2. Self-energy

The laser field is described by a plane wave with wavevector k:

$$A_\mu(x) = ae_{i\mu}a_i(kx) \qquad ke_i = 0 \qquad e_ie_j = -\delta_{ij}$$

where a summation convention is understood for repeated polarization indices.

For the calculation of the self-energy and the propagator the technique of the Volkov transform turns out to be useful. This amounts to representing any quantity $F(x, x')$ referring to an electron (as e.g. the self-energy Σ, the uncorrected propagator G or the corrected one G') sandwiched between Volkov wavefunctions $E(x|p)$ in the following way:

$$F(x,x') = \frac{1}{(2\pi)^4}\int \mathrm{d}^4p\, \mathrm{d}^4p'\, E(x|p)\tilde{F}(p,p')\bar{E}(x'|p') \tag{1}$$

and computing $\tilde{F}$ rather than F. The functions E fulfil the Dirac-type equation

$$\Pi\!\!\!/(x)E(x|p) := \gamma_\nu(\mathrm{i}\partial^\nu - \epsilon A^\nu)E(x|p) = E(x|p)p\!\!\!/. \tag{2}$$

Hence $E(x|p)\psi_p$ with a free spinor ψ_p, such that $(p\!\!\!/-\kappa)\psi_p = 0$ solves the Dirac equation. The E can be continued off the mass shell for p so that they are orthogonal and complete

$(\bar{E}(x|p) = \gamma^0 E(x|p)^+ \gamma^0)$:

$$\begin{aligned} \frac{1}{(2\pi)^4} \int \bar{E}(x|p) E(x|p')\, \mathrm{d}^4 x &= \delta(p-p') \\ \frac{1}{(2\pi)^4} \int E(x|p) \bar{E}(x'|p)\, \mathrm{d}^4 p &= \delta(x-x'). \end{aligned} \tag{3}$$

In addition to calculational simplicity, the Volkov transform has some general advantages in comparison with the Fourier transform, which is its limit in absence of the external field. For instance, it does not depend on the gauge chosen for the latter, and we may use the conventional Feynman rules and propagators familiar from the theory without external field. The only difference is, that we have to use instead of the vertex $\gamma_\mu \delta(p-p'+q)$ the modified expression:

$$v_\mu(p, p'|q) = \frac{1}{(2\pi)^4} \int \mathrm{d}^4 x\, \bar{E}(x|p') \gamma_\mu E(x|p)\, \mathrm{e}^{\mathrm{i}qx}. \tag{4}$$

Explicit formulae for E and v_μ and more details on Volkov transforms have been discussed in an earlier paper (Mitter 1975, to be referred to as M).

For the self-energy we have to consider the expression (M(VII.11))

$$\tilde{\Sigma}(p, p') = \frac{\mathrm{i}\epsilon^2}{(2\pi)^4} \int \mathrm{d}^4 q\, \mathrm{d}^4 q'\, v_\mu(p, q|q')(\not{q} - \kappa)^{-1} v_\nu(q, p'|-q') D^{\mu\nu}(q') + \ldots. \tag{5}$$

Here $D^{\mu\nu}$ is the free photon propagator in momentum space. We shall extract the self-energy in the absence of the external field $\Sigma_0(p)$ writing:

$$\tilde{\Sigma}(p, p') = \Sigma_0(p)\delta(p-p') + \tilde{\Sigma}_L(p, p'). \tag{6}$$

The second term $\tilde{\Sigma}_L$ does not contain any (infrared or ultraviolet) divergencies, so that no regularization procedure is needed for this term. At a first glance the convergence of the part depending on the external field seems surprising, since a perturbation expansion of $\Sigma(x, x')$ with respect to the coupling to the latter field contains the vertex part, which diverges logarithmically. That things are different and better for the Volkov transform, is due to gauge invariance. Since $\Sigma(x, x')$ diverges linearly, an expansion at small distances will contain two divergent contact terms proportional to the δ function and its derivative. Because of gauge invariance the derivative must appear in the combination Π_μ, so that we have

$$\Sigma(x, x') = (A + B\not{\Pi}(x))\, \delta(x-x') + \text{finite terms}. \tag{7}$$

The divergent constants A, B could only depend on the external field via invariant combinations formed with $F^{\mu\nu}$. Since, however, all these combinations are zero in our case, A and B cannot depend on the laser field at all. Because of equation (2) the Volkov transform of the contact term in (7) yields just the divergent part of the free self-energy and $\tilde{\Sigma}_L$ is indeed finite. Thus the renormalization problem reduces to the corresponding one in the absence of the laser field. We shall therefore assume henceforth, that $\Sigma_0(p)$ is already properly renormalized, so that κ is the physical mass and $Z_2 = 1$. (In the language of Schwinger's source theory this removal of the contact terms is called propagator normalization, cf the corresponding treatment for a constant magnetic field (Tsai *et al* 1973).) Then $\Sigma_0(p)$ is an expression of the form

$$\Sigma_0(p) = \kappa B_f^{(0)}(p^2) + \not{p} B_f^{(1)}(p^2). \tag{8}$$

The functions $B_f^{(i)}$ are proportional to α and fulfil the relations

$$\begin{aligned} &B_f^{(0)}(\kappa^2)+B_f^{(1)}(\kappa^2)=0 \\ &B_f^{(1)}(\kappa^2)+2\kappa^2\frac{\mathrm{d}}{\mathrm{d}p^2}(B_f^{(0)}+B_f^{(1)})|_{p^2=\kappa^2}=0. \end{aligned} \tag{9}$$

The explicit form of $B_f^{(i)}$ depends on the gauge chosen for the quantized radiation field and can be found in the literature, cf Bjørken and Drell (1965, § 8) (Feynman gauge) or Breitenlohner and Mitter (1968, § 6) (Yennie–Fried gauge; in this case the infrared divergences are absent to this order in α). We shall not need these expressions for our purposes.

The calculation of $\tilde{\Sigma}_L$ is somewhat lengthy but standard, and we shall omit the details. We shall use the Feynman gauge, for which the photon propagator is given by

$$D_{\mu\nu}(q)=\mathrm{i}\int_0^\infty \mathrm{d}t\ \mathrm{e}^{\mathrm{i}t(q^2+\mathrm{i}\epsilon)}g_{\mu\nu}. \tag{10}$$

In fact, it would also be possible to use any other covariant gauge, i.e. to replace $g^{\mu\nu}$ by $g^{\mu\nu}-\mathrm{i}tg^{\mu}q^{\nu}$ with an arbitrary constant G, since the gauge term can be calculated easily using the relation (M(V.6)) for the vertex v_μ. We have, however, not reached any apparent simplification by choosing any $G\neq 0$, so that the Feynman gauge seems preferable. For the calculation the use of light-like coordinates is very convenient, since then as many integrations as possible are trivial. The field dependent part can be expressed in terms of threefold integrals of elementary functions. We shall give the result in terms of some functions (L_i, M_i, R, $\hat{R}$, T) of two variables η, ζ, which have been used before in calculating vacuum polarization effects (Becker and Mitter 1975) and are extensively discussed in appendix 1 of this reference.

In addition to these functions we shall use the abbreviations

$$y=y_0[1+(T/\kappa^2)], \qquad y_0=\zeta\kappa^2/2|pk| \tag{11}$$

and the integrals

$$K_{nm}(y)=\int_0^\infty \mathrm{d}\rho\,\rho^n(1+\rho)^{-m}\,\mathrm{e}^{-\mathrm{i}y\rho} \tag{12}$$

which can be expressed in terms of the exponential integral of imaginary argument, e.g. $K_{0m}(y)=\exp(\mathrm{i}y)E_m(\mathrm{i}y)$, cf Abramowitz and Stegun (1965). The expression for the self-energy then becomes:

$$\begin{aligned} \tilde{\Sigma}_L(p,p')=\delta(p_v-p'_v)\delta(p_i-p'_i)(2\pi\sqrt{2}\omega)^{-1}\int_{-\infty}^{\infty}\mathrm{d}\eta\,\exp[\mathrm{i}\eta(p_u-p'_u)/\sqrt{2}\omega] \\ \times[\kappa B^{(0)}+\tfrac{1}{2}(\not{p}+\not{p}')B^{(1)}+\not{k}B^{(2)}+\mathrm{i}\gamma_5\not{k}B^{(3)}+(C_i^{(0)}+\not{k}C_i^{(1)}+\mathrm{i}\gamma_5C_i^{(2)})\hat{\not{e}}_i]. \end{aligned} \tag{13}$$

Here we have used the vectors

$$\hat{e}_i^\mu=e_i^\mu-\frac{pe_i}{pk}k^\mu \qquad i=1,2 \tag{14}$$

which fulfil

$$p\hat{e}_i=k\hat{e}_i=0, \qquad \hat{e}_i\hat{e}_j=-\delta_{ij}.$$

The coefficients in (13) read

$$(C^{(k)}, B^{(k)}) = \frac{\alpha}{2\pi}\int_0^\infty \mathrm{d}\zeta (Q^{(k)}, P^{(k)}/\zeta) \exp\left(\mathrm{i}y_0 \frac{pp'-\kappa^2}{\kappa^2}\right) \tag{15}$$

with

$$\begin{aligned} &P^{(0)} = 2(K_{02}(y) - K_{02}(y_0)) \qquad P^{(1)} = -(K_{03}(y) - K_{03}(y_0)) \\ &P^{(2)} = \zeta/(2pk)(-RK_{12}(y) + 2\zeta L_i^2 K_{13}(y) + 4\zeta M_i^2 K_{02}(y)) \\ &P^{(3)} = \mathrm{i}\hat{R}\zeta/(2|pk|)(K_{12}(y) + K_{13}(y)) \\ &Q_i^{(0)} = L_i K_{13}(y) \qquad Q_i^{(1)} = 2\kappa/|pk| M_i K_{02}(y) \\ &Q_i^{(2)} = -pk/|pk| \epsilon_{ij} M_j (K_{03}(y) + K_{02}(y)). \end{aligned} \tag{16}$$

The form (16) holds for arbitrary plane wave fields. One may check, that the expression has the correct behaviour under charge conjugation

$$\tilde{\Sigma}(p, p'|A) = C\Sigma(-p', -p|-A)^{\mathrm{T}} C^{-1}. \tag{17}$$

The symmetry property (17) holds also for the corrected propagator and is valid to arbitrary order in α.

For a constant, crossed field (e.g. $a_1 = a_2 = kx$) the self-energy becomes diagonal and we recover the result given by Ritus (1972) for this case. For circular polarization

$$a_1 = \cos kx, \qquad a_2 = -\sin kx$$

the η-dependence of L_i and M_i is elementary, all bilinear combinations of these functions as well as T, R, $\hat{R}$ do not depend on η and the integral on η reduces to δ functions. The result has the form

$$\tilde{\Sigma}(p, p') = S^{(0)}(p)\delta(p-p') + S^{(+)}(p)\delta(p-p'+k) + S^{(-)}(p)\delta(p-p'-k) \tag{18}$$

with

$$S^{(0)}(p) = \Sigma^{(0)}(p) + \Sigma_L^{(0)}(p). \tag{19}$$

This general structure results to arbitrary order in α (see appendix 2). In first approximation we have

$$\Sigma_L^{(0)} = \kappa B^{(0)} + \not{p} B^{(1)} + \not{k} B^{(2)} + \mathrm{i}\gamma_5 \not{k} B^{(3)} \tag{20}$$

with the same functions B as given above: of course now the corresponding functions R etc for circular polarization have to be used in $P^{(k)}$ (cf formulae (A1.4)–(A1.6) of Becker and Mitter 1975). The non-diagonal part is given by

$$S^{(\pm)}(p) = (C_\pm^{(0)} \pm \not{k} C_\pm^{(1)} + \mathrm{i}\gamma_5 C_\pm^{(2)})\hat{\not{e}}_\pm \tag{21}$$

where we have

$$\hat{e}_\pm = (\hat{e}_1 \pm \mathrm{i}\hat{e}_2)/\sqrt{2}$$

and

$$C_\pm^{(k)}(p) = \frac{\alpha}{2\pi}\int_0^\infty \mathrm{d}\zeta\, Q^{(k)} \exp[\mathrm{i}y_0(p^2 \pm pk - \kappa^2)/\kappa^2] = C_\mp^{(k)}(p \pm k). \tag{22}$$

The $Q^{(k)}$ contained in this expression are obtained from the $Q_i^{(k)}$ used above by the replacement

$$L_i \to -L/\sqrt{2}, \qquad M_i \to -\mathrm{i}M/\sqrt{2}, \qquad \epsilon_{ik} M_k \to M/\sqrt{2}. \tag{23}$$

3. Corrected propagator

Now we shall evaluate the corrected propagator for circular polarization. We have to solve the equation

$$\int G'^{-1}(x, x'')\,\mathrm{d}^4x''\, G'(x'', x') = \delta(x - x') \tag{24}$$

with

$$G'^{-1}(x, x') = (\not{\Pi}(x) - \kappa)\delta(x - x') - \Sigma(x, x'). \tag{25}$$

The solution is most easily obtained, if we represent G' according to (1) and invert first the corresponding equation for $\tilde{G}'$, which reads (cf M(VII.10))

$$\tilde{G}'^{-1}(p, p') = (\not{p} - \kappa)\delta(p - p') - \tilde{\Sigma}(p, p'). \tag{26}$$

Since $\tilde{\Sigma}$ is 'almost diagonal' for circular polarization, the inversion reduces to a set of algebraic equations. These turn out to be particularly simple, if we observe, that the propagator $\tilde{G}'$ has the same diagonality properties as its inverse

$$\tilde{G}'(p, p') = G^{(0)}(p)\delta(p - p') + G^{(+)}(p)\delta(p - p' + k) + G^{(-)}(p)\delta(p - p' - k). \tag{27}$$

This structure holds, as the corresponding one for $\tilde{\Sigma}$, to arbitrary order in α and is due to the additional symmetry encountered for circular polarization as mentioned in (M, § V).

The parts entering (27) are then obtained in terms of the corresponding ones in (18) by

$$\begin{aligned} G^{(0)}(p) = [\not{p} - \kappa - S^{(0)}(p) - S^{(+)}(p)(\not{p} + \not{k} - \kappa - S^{(0)}(p + k))^{-1}S^{(-)}(p + k) \\ - S^{(-)}(p)(\not{p} - \not{k} - \kappa - S^{(0)}(p - k))^{-1}S^{(+)}(p - k)]^{-1} \end{aligned} \tag{28}$$

$$G^{(\pm)}(p) = (\not{p} - \kappa - S^{(0)}(p))^{-1}S^{(\pm)}(p)G^{(0)}(p \pm k). \tag{29}$$

These formulae hold again to arbitrary order in α and contain only algebraic inversions. The actual calculation, which results, if we insert the second order expressions (19), (21), is rather tedious, but does not present any principal problem. An outline of the calculation is given in appendix 1. The diagonal part has the structure

$$G^{(0)}(p) = Z_+(p)\,W_-/D_+(p) + Z_-(p)\,W_+/D_-(p) \tag{30}$$

where we have

$$Z_\pm(p) = S \mp T_1 - \not{p}V_1 - \gamma_5\not{p}A_1 - \not{k}V_2 - \gamma_5\not{k}A_2 \tag{31}$$

$$W_\pm = \tfrac{1}{2}(1 \pm \tfrac{1}{2}[\not{e}_+, \not{e}_-]). \tag{32}$$

The functions S, T_1 etc entering (31) and the denominators $D_\pm$ are invariants, which can be expressed in terms of the functions (15). Explicit expressions are given in appendix 1. All functions contain terms up to second order in α. This is evident from (28), since $S^{(\pm)}$ is proportional to α. The off-diagonal parts have the form

$$G^{(\pm)}(p) = \frac{Z_0(p)S^{(\pm)}(p)Z_\mp(p \pm k)}{N_\pm(p)D_\mp(p \pm k)} = \frac{Z_0(p)S^{(\pm)}(p)Z_\mp(p \pm k)}{N_\mp(p \pm k)D_\pm(p)}. \tag{33}$$

Here Z_0 is obtained from $Z_\pm$ by dropping the terms of order α^2 (both functions are equal in this limit) and $N_\pm$ is obtained in the same fashion from $D_\pm$. Explicit expressions are again given in appendix 1.

The poles of $\tilde{G}'$, i.e. the zeros of the denominator functions, are closely related to the quasi-levels of the electron in the laser field. The exact determination of these zeros is very difficult, since the corresponding functions are quite complicated. Also the whole propagator (30) is too complicated to be used in further calculations. For most applications very fine details will not matter and we shall be content with approximate forms of the propagator.

The most obvious approximation is an expansion (separately in the numerator and the denominator) in powers of α. This may turn out to be inadequate, however, if α is not the smallest parameter, i.e. if $2pk/\kappa^2 \ll \alpha$. We shall return to this point later. To order α, the numerator of the diagonal part becomes

$$Z_\pm \simeq Z_0 = -\kappa(1+B_f^{(0)}+B^{(0)}) - \not{p}(1-B_f^{(1)}-B^{(1)}) + \not{k}B^{(2)} + \mathrm{i}\gamma_5\not{k}B^{(3)} \tag{34}$$

and the denominators read

$$\begin{aligned} D_\pm(p) \simeq N_\pm(p) = \kappa^2(1+B_f^{(0)}+B^{(0)})^2 - (1-B_f^{(1)}-B^{(1)}) \\ \times[p^2(1-B_f^{(1)}-B^{(1)}) - 2pk(B^{(2)} \pm \mathrm{i}B^{(3)})]. \end{aligned} \tag{35}$$

Now we extract a factor $(1-B_f^{(1)}-B^{(1)})^2$ from the denominator. Expanding in powers of α we obtain for the expression

$$R(p) = -Z_0(p)/(1+B_f^{(1)}-B^{(1)})^2 \tag{36}$$

the following approximate form:

$$R(p) = \kappa(1+B_f^{(0)}+B_f^{(1)}+B^{(0)}+B^{(1)}) + \not{p} - \not{k}B^{(2)} - \mathrm{i}\gamma_5\not{k}B^{(3)} + \mathrm{O}(\alpha^2). \tag{37}$$

A corresponding approximation in the remainder of the denominator yields an approximate form for the propagator, which is accurate up to linear terms in α. We have

$$G^{(0)}(p) = R(p)(W_+/Y_-(p) + W_-/Y_+(p)) \tag{38}$$

$$G^{(\pm)}(p) = (\kappa + \not{p})S^{(\pm)}(p)(\kappa + \not{p} \pm \not{k})/(Y_\pm(p)\,Y_\mp(p \pm k)) \tag{39}$$

with

$$Y_\pm(p) = p^2 - \kappa^2 - \kappa^2 M_\pm(p) \tag{40}$$

$$M_\pm(p) = 2(B_f^{(0)}+B_f^{(1)}+B^{(0)}+B^{(1)}) + (2pk/\kappa^2)(B^{(2)} \pm \mathrm{i}B^{(3)}). \tag{41}$$

Since all B are proportional to α, the zeros of the denominators will result from an equation of the type $p^2-\kappa^2 = \alpha f(p^2, pk)$. We can therefore approximate p^2 by κ^2 in the argument of f. Then the poles are determined by

$$p^2 \simeq \kappa^2(1+M_\pm(p^2=\kappa^2)) \tag{42}$$

with

$$M_\pm(p^2=\kappa^2) = 2(B^{(0)}+B^{(1)}) + \frac{2pk}{\kappa^2}(B^{(2)} \pm \mathrm{i}B^{(3)})|_{p^2=\kappa^2} \tag{43}$$

since the B_f drop out because of (9). The functions M depend on the laser intensity parameter

$$\nu = \epsilon a/\kappa \tag{44}$$

and the variable

$$\rho = 2pk/\kappa^2 \tag{45}$$

and must be determined numerically. Equations (42) and (43) agree with the results of Baier *et al* (1975) obtained by mass shell averaging the mass operator. The imaginary part is related to the total cross section of high intensity Compton scattering (Brown and Kibble 1964, Nikishov and Ritus 1964, Goldman 1964). The imaginary part of (43) can be rewritten to yield the well known series of squares of Bessel functions (Baier *et al* 1975).

The mass correction $\Delta\kappa/\kappa = \frac{1}{2}M_\pm$ for $\nu^2 = 1$ and $0{\cdot}1 \leqslant \rho \leqslant 3$ is plotted in figure 1. For smaller values of ρ the poles almost coincide. For $\rho \leqslant 0{\cdot}01$ we can apply the following approximate formulae for the imaginary part which are correct up to a few per cent:

$$\mathrm{Im}(\Delta\kappa/\kappa) = \begin{cases} -0{\cdot}145\alpha\rho(\nu^2)^{0{\cdot}93} & 0{\cdot}1 \leqslant \nu^2 \leqslant 1{\cdot}0 \\ -\frac{1}{6}\alpha\rho\nu^2 & \nu \leqslant 0{\cdot}1. \end{cases} \tag{46}$$

Figure 1 and equation (46) exhibit the negative sign of the imaginary part of the mass correction, as it must be, since the electron is no longer stable but can emit photons via high intensity Compton scattering. Equation (46) differs remarkably from the constant crossed field case where only the combined variable $\rho\nu$ occurs.

The real part of $\Delta\kappa/\kappa$ is seen from figure 1 to vary as ρ^2. Hence, for small values of ρ such that $\rho \leqslant \alpha$, our approximation breaks down since we have retained $O(\rho^2\alpha)$ but neglected $O(\rho\alpha^2)$, which becomes dominant. The determination of the real part of the poles becomes extremely complicated in this case and requires the complete formulae as given in the appendix. For $\rho \ll \alpha$ the zeros of $D_\pm = 0$ are to lowest order of ρ given by

$$p^2 \cong \kappa^2(1 + \mathrm{Re}\,\hat{M}_\pm(p^2 = \kappa^2) + \mathrm{i}\,\mathrm{Im}\,M_\pm(p^2 = \kappa^2)) \tag{47}$$

where now

$$\hat{M}_\pm(p^2 = \kappa^2) = \mp\frac{4}{pk}\left(C_\pm^{(2)} + \mathrm{i}\frac{pk}{\kappa}C_\pm^{(1)}\right)\bigg|_{p^2=\kappa^2} \tag{48}$$

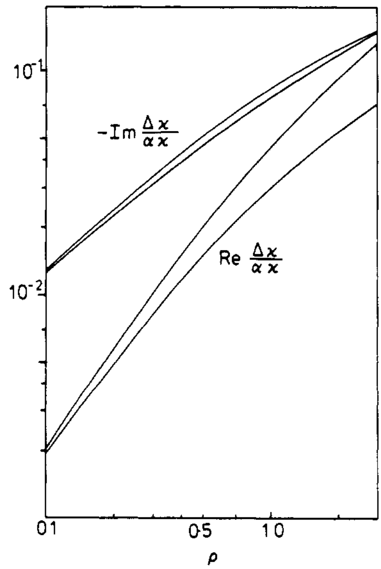

Figure 1. Real and imaginary part of the mass shift for $\nu^2 = 1$ and $0{\cdot}1 \leqslant \rho \leqslant 3$.

and $M_\pm$ is still given by equation (43). An expansion in powers of ν^2 which actually can be used up to $\nu^2 \leqslant 1$ reads

$$\mathrm{Re}\,\frac{\Delta\kappa}{\kappa} = \pm\frac{\alpha^2\nu^2\rho}{32} \qquad (\rho \ll \alpha). \tag{49}$$

This is extremely small and we shall not pursue this case any longer. For all applications, the $\mathrm{O}(\alpha)$ approximate propagator as obtained by using equations (38)–(41) with equation (43) should prove sufficient: it incorporates a good approximation to the imaginary part of the poles and a fair one to the real part because the very small values of (48) are (wrongly) replaced by even smaller ones. In principle, however, it is remarkable that the mass shell average of the mass operator does not, in every case, yield the correct mass shift. Also the field zero limiting value $\alpha/2\pi$ of the anomalous magnetic moment which is related to the splitting of the real part of $M_\pm$, results correctly from equation (48) and not from equation (43).

4. Level structure

The quasi-levels are determined by the poles of the Fourier transform rather than the Volkov transform of the propagator. These transforms are related by

$$\begin{aligned}\hat{G}'(q,q') &= \frac{1}{(2\pi)^4}\int \mathrm{d}^4x\,\mathrm{d}^4x'\,G'(x,x')\,\mathrm{e}^{\mathrm{i}(qx-q'x')} \\ &= \int \mathrm{d}^4p\,\mathrm{d}^4p'\,\hat{E}(q|p)\tilde{G}'(p,p')\hat{\bar{E}}(q',p').\end{aligned} \tag{50}$$

Here $\hat{E}$ is the Fourier transform of E

$$\hat{E}(q|p) = \frac{1}{(2\pi)^4}\int \mathrm{d}^4x\,E(x|p)\,\mathrm{e}^{\mathrm{i}qx}. \tag{51}$$

For circular polarization we obtain the result

$$\hat{E}(q|p) = \mathrm{e}^{-\mathrm{i}\phi_0}\delta(p_v-q_v)\delta(p_i-q_i)\sum_{n=-\infty}^{\infty} K_n(p_v,p_i)\delta\!\left(p_u - nk_u + \frac{\epsilon^2a^2}{2p_v} - q_u\right) \tag{52}$$

where we have used the light-like coordinates and

$$K_n = J_n\,\mathrm{e}^{\mathrm{i}n\tilde{\rho}} - \frac{\epsilon a}{2\sqrt{2}p_v}\gamma_v(\gamma + J_{n-1}\,\mathrm{e}^{\mathrm{i}(n-1)\tilde{\rho}} + \gamma_- J_{n+1}\,\mathrm{e}^{\mathrm{i}(n+1)\tilde{\rho}}). \tag{53}$$

The argument of the Bessel functions J_n is $p\epsilon a/pk$ and we have

$$p_1 = p\cos\tilde{\rho}, \qquad p_2 = p\sin\tilde{\rho}.$$

The corresponding form in coordinate space (including the phase ϕ_0) has been given in (M(III.32)) for the function on shell. Here we need the corresponding off-shell form, which results from (M(III.32)) by $p_u^{\mathrm{eff}} \to p_u$. If we use these results and the propagator

(27), the integrations can be performed trivially and we obtain

$$\hat{G}'(q,q')=\sum_{r=-\infty}^{\infty}\delta(q-q'+rk)\sum_{n=-\infty}^{\infty}K_n(q)$$
$$\times[G^{(0)}(\hat{q}_n)\bar{K}_{n-r}(q)+G^{(+)}(\hat{q}_n)\bar{K}_{n-r+1}(q)+G^{(-)}(\hat{q}_n)\bar{K}_{n-r-1}(q)] \tag{54}$$

with

$$\hat{q}_n=q+[n-(\epsilon^2a^2/2qk)]k. \tag{55}$$

We observe, that the propagator in momentum space is not diagonal at all, as is known from the theory without radiative corrections (Reiss and Eberly 1966). Because of

$$\hat{q}_n^2-\kappa^2=(q+nk)^2-\kappa^2(1+\nu^2), \qquad \hat{q}_nk=qk$$

the poles given by equation (42) correspond now to modified quasi-levels, determined by

$$(q+nk)^2=\kappa^2(1+\nu^2+M_\pm). \tag{56}$$

The real part of M causes a shift and the imaginary part a finite width of these levels. As long as $M_\pm$ is small, the levels remain narrow.

Appendix 1. The complete propagator

In this appendix, we shall give the complete result of summing the self-energy calculated in § 2. Approximations have been discussed in § 3. In order to evaluate equations (28) and (29) we need the inverse of expressions of the form

$$A=s+v_1\not{p}+v_2\not{k}+a_1\gamma_5\not{p}+a_2\gamma_5\not{k}+t_1\tfrac{1}{2}[\hat{\not{e}}_+,\hat{\not{e}}_-]. \tag{A.1}$$

It is given by

$$A^{-1}=\gamma_5A\gamma_5(W_+/d_-+W_-/d_+) \tag{A.2}$$

which has the structure displayed in equation (30), and we have

$$d_\pm=(s\mp t_1)^2+(2pk(a_2\pm v_2)+p^2(a_1\pm v_1))(a_1\mp v_1). \tag{A.3}$$

The functions which appear in (31) and (33) are given by

$$\begin{aligned}
S\pm T_1&=-\kappa I_0(p)\mp 4pkI_1(p_\mp)C_\mp^{(1)}(C_\mp^{(0)}\pm \mathrm{i}C_\mp^{(2)})/N_\pm(p_\mp)-2\kappa I_0(p_\pm)(C_\mp^{(0)2}+C_\mp^{(2)2})/N_\pm(p_\mp)\\
V_1\pm A_1&=I_1(p)+2I_1(p_\mp)(C_\mp^{(0)}\pm \mathrm{i}C_\mp^{(2)})^2/N_\pm(p_\mp)\\
V_2\pm A_2&=-B^{(2)}(p)\mp \mathrm{i}B^{(3)}(p)+2\Big[2pkI_1(p_\pm)C_\pm^{(1)2}\mp 2\kappa I_0(p_\pm)C_\pm^{(1)}(C_\pm^{(0)}\pm \mathrm{i}C_\pm^{(2)})\\
&\quad-(C_\pm^{(0)}\pm \mathrm{i}C_\pm^{(2)})^2\Big(B^{(2)}(p_\pm)\mp \mathrm{i}B^{(3)}(p_\pm)-\frac{p_\pm^2}{2pk}I_1(p_\pm)\Big)\Big](N_\mp(p_\pm))^{-1}\\
&\quad-p^2I_1(p_\mp)(C_\mp^{(0)}\pm \mathrm{i}C_\mp^{(2)})^2/(pkN_\pm(p_\mp))
\end{aligned} \tag{A.4}$$

where $p_\pm=p\pm k$ and $I_0(p)=1+B_f^{(0)}+B^{(0)}$, $I_1(p)=1-B_f^{(1)}-B^{(1)}$. $D_\pm$ is obtained from

$d_\pm$ by replacing s, t_1 etc by S, T_1 etc and reads explicitly

$$\begin{aligned}D_\pm(p)N_\mp(p_\pm) &= N_\pm(p)N_\mp(p_\pm) + 4(C_\pm^{(0)2} + C_\pm^{(2)2})^2 + 4\kappa^2 I_0(p)I_0(p_\pm)(C_\pm^{(0)2} + C_\pm^{(2)2}) \\ &\quad - 8(pk)^2 I_1(p)I_1(p_\pm)C_\pm^{(1)2} \mp 8\kappa pk C_\pm^{(1)}[I_1(p_\pm)I_0(p)(C_\pm^{(0)} \mp iC_\pm^{(2)}) \\ &\quad + I_1(p)I_0(p_\pm)(C_\pm^{(0)} \pm iC_\pm^{(2)})] + 4pk[I_1(p)(B^{(2)}(p_\pm) \mp iB^{(3)}(p_\pm))(C_\pm^{(0)} \pm iC_\pm^{(2)})^2 \\ &\quad + I_1(p_\pm)(B^{(2)}(p) \mp iB^{(3)}(p))(C_\pm^{(0)} \mp iC_\pm^{(2)})^2] \\ &\quad - 2I_1(p)I_1(p_\pm)[p^2(C_\pm^{(0)} \mp iC_\pm^{(2)})^2 + p_\pm^2(C_\pm^{(0)} \pm iC_\pm^{(2)})^2] \end{aligned} \tag{A.5}$$

$$N_\pm(p) = \kappa^2 I_0(p)^2 - I_1(p)[p^2 I_1(p) - 2pk(B^{(2)} \pm iB^{(3)})]. \tag{A.6}$$

From their definition we infer the following behaviour of the relevant functions upon the substitution $p \to -p$:

$$\begin{array}{lll} B^{(i)}(p) = B^{(i)}(-p) & i = 0, 1, 3 & B^{(2)}(p) = -B^{(2)}(-p) \\ C_\pm^{(i)}(p) = C_\mp^{(i)}(-p) & i = 0, 1 & C_\pm^{(2)}(p) = -C_\mp^{(2)}(-p) \end{array} \tag{A.7}$$

and consequently

$$N_\pm(p) = N_\mp(-p) \qquad D_\pm(p) = D_\mp(-p). \tag{A.8}$$

From (A.7), (A.8) and (22) we obtain

$$N_\pm(p)D_\pm(-p_\pm) = N_\mp(p_\pm)D_\pm(p). \tag{A.9}$$

Equations (A.8) and (A.9) give some insight into the general structure of the poles of the propagator. We assume the zeros of $N_\pm(p)$ and $D_\pm(p)$ to be different. If $p^2 = f(pk)$ is a zero of $D_+(p) = 0$, then $(p+k)^2 = f(-pk)$ is another zero because of (A.9). Accordingly, if $p^2 = g(pk)$ is a zero of $D_-(p) = 0$, $(p-k)^2 = g(-pk)$ is another one. Because of (A.8), $g(pk) = f(-pk)$. So, finally, the zeros of $D_+(p)$ are given by

$$p^2 = f(pk) \qquad (p+k)^2 = f(-pk)$$

and those of $D_-(p)$ by

$$p^2 = f(-pk) \qquad (p-k)^2 = f(pk).$$

Hence, the poles of the propagator exhibit two independent branches, $p^2 = f(\pm pk)$, corresponding to opposite spin orientations. As α vanishes, the zeros given by $(p \pm k)^2 = f(\mp pk)$ cancel because $N_\mp(p_\pm)$ drops out from (A.5) and only those given by $N_\pm(p) = 0$ survive which we discussed above in § 3.

One might conclude from equations (A.4) that in addition to the poles just discussed the zeros of $N_\pm(p_\mp)$ give rise to further poles. This is not the case. Evaluating the products $Z_\pm W_\mp$ in equation (30) it turns out that the functions S, T_1, A, V appear in combinations such that either $N_+(p_-)^{-1}$ or $N_-(p_+)^{-1}$ remains present, which then combines with $D_-(p)$ or $D_+(p)$ to give equation (A.5). The same is also true for the off-diagonal part (33).

Appendix 2. Proof of diagonality properties of the propagator

A circularly polarized plane wave field is invariant with respect to the following Poincaré transformations (a, L): $x \to x' = L(x+a)$ (Richard 1972) ($k_\mu = \sqrt{2}\omega n_\mu$,

$n\hat{n}=1$, $\hat{n}^2=0$, $\hat{n}e_i=0$)

$$a^\mu=\epsilon n^\mu+\epsilon_i e_i^\mu-\frac{1}{\sqrt{2}}\left(\hat{\epsilon}+\frac{2l\pi}{\omega}\right)\hat{n}^\mu \tag{A.10}$$

$$\begin{aligned}L^{\mu\nu}=g^{\mu\nu}+\alpha\alpha^* n^\mu n^\nu+\alpha\,\mathrm{e}^{-\mathrm{i}\omega\hat{\epsilon}}e_+^\mu n^\nu-\alpha n^\mu e_+^\nu+\alpha^*\,\mathrm{e}^{\mathrm{i}\omega\hat{\epsilon}}\,e_-^\mu n^\nu-\alpha^* n^\mu e_-^\nu\\ -e_-^\mu e_+^\nu(\mathrm{e}^{\mathrm{i}\omega\hat{\epsilon}}-1)-e_+^\mu e_-^\nu(\mathrm{e}^{-\mathrm{i}\omega\hat{\epsilon}}-1)\end{aligned} \tag{A.11}$$

where l is an integer, ϵ, ϵ_i, $\hat{\epsilon}$ are real and α is complex. The transformations specified by (A.10) and (A.11) allow for arbitrary translations which are, however, accompanied by a compensating rotation. It is this property which leads to an almost diagonal propagator and mass operator.

The electron field operator transforms according to

$$U(a,L)\psi^A(x)U^{-1}(a,L)=S^{-1}(L)\psi^{A'}(L(x+a)). \tag{A.12}$$

The upper index A denotes the dependence of the field operator on the vector potential. Hence $A'_\mu=L_\mu^\nu A_\nu$, and A and A' differ by a gauge transformation (cf (A.19)). $S(L)$ is the usual transformation matrix which is determined by the antisymmetric first order part of $L_{\mu\nu}$

$$S(L)=\exp\left(-\frac{\mathrm{i}}{4}\,l_{\mu\nu}\sigma^{\mu\nu}\right)$$

$$l_{\mu\nu}=\alpha(e_+^\mu n^\nu-n^\mu e_+^\nu)+\alpha^*(e_-^\mu n^\nu-n^\mu e_-^\nu)-\mathrm{i}\omega\hat{\epsilon}(e_-^\mu e_+^\nu-e_+^\mu e_-^\nu). \tag{A.13}$$

An arbitrary vector b^μ is invariant with respect to special transformations (A.11) with

$$\alpha=\frac{\omega b_-}{kb}(\mathrm{e}^{\mathrm{i}\omega\hat{\epsilon}}-1)\qquad b_\pm=-\sqrt{2}e_{\pm\mu}b^\mu. \tag{A.14}$$

From now on we fix α according to (A.14) with $b_\mu=p_\mu$ the momentum vector. Since p'_μ and p_μ differ by a multiple of the wavevector k_μ and $L_{\mu\nu}k^\nu=k_\mu$, p'_μ is invariant as well. If we insert (A.14) into (A.13) (to first order in $\hat{\epsilon}$ as required by consistency) $l_{\mu\nu}$ takes the simple form

$$l^{\mu\nu}=\mathrm{i}\omega\hat{\epsilon}(\hat{e}_+^\mu\hat{e}_-^\nu-\hat{e}_-^\mu\hat{e}_+^\nu) \tag{A.15}$$

with

$$\hat{e}_\pm^\mu=e_\pm^\mu+(b_\pm/\sqrt{2}\,bk)k^\mu$$

as in equation (14). Hence

$$\begin{aligned}S(L)&=\exp(\tfrac{1}{4}\mathrm{i}\omega\hat{\epsilon}[\hat{e}\!\!\!/_+,\hat{e}\!\!\!/_-])=\cos\tfrac{1}{2}\omega\hat{\epsilon}+\tfrac{1}{2}\mathrm{i}[\hat{e}\!\!\!/_+,\hat{e}\!\!\!/_-]\sin\tfrac{1}{2}\omega\hat{\epsilon}\\ S^{-1}(L)&=S(L)|_{\hat{\epsilon}\to-\hat{\epsilon}},\qquad \overline{S^{-1}}(L)=S(L).\end{aligned} \tag{A.16}$$

We need the following properties of $S(L)$:

$$S(L)E(x|p)S^{-1}(L)=E(L(x+a)|p)\exp\left(\mathrm{i}pa-\frac{\mathrm{i}\epsilon}{2pk}\int_\xi^{\xi+ka}\mathrm{d}\xi'(-2pA+\epsilon A^2)\right) \tag{A.17}$$

$$S(L)\hat{e}\!\!\!/_\pm S^{-1}(L)=\hat{e}\!\!\!/_\pm\exp(\mp\mathrm{i}\omega\hat{\epsilon}). \tag{A.18}$$

The vector potential $A^\mu(x)=[e_+^\mu\exp(\mathrm{i}\xi)+e_-^\mu\exp(-\mathrm{i}\xi)]a/\sqrt{2}$ transforms as

$$L^{\mu\nu}A_\nu(x)=A^\mu(L(x+a))+\partial^\mu(\Lambda(L(x+a))-\Lambda(x)) \tag{A.19}$$

with a gauge function

$$\Lambda(x) = \frac{ia}{2pk}(p_- \, e^{-i\xi} - p_+ \, e^{i\xi}) = \frac{1}{2pk}\int^{\xi} d\xi''(-2pA(\xi'')). \tag{A.20}$$

We can now investigate the behaviour of the Volkov transform of the propagator with respect to the transformations (a, L) specified by (A.14). Using (A.12) and (A.17) we obtain

$$\begin{aligned} i\tilde{G}'(p,p') &= \frac{1}{(2\pi)^4}\int d^4x \, d^4x' \, \bar{E}(x|p)\langle 0|T\psi^{A'}(x)\bar{\psi}^{A'}(x')|0\rangle E(x'|p') \\ &= \frac{1}{(2\pi)^4}\int d^4x \, d^4x' \exp\left[-i(p-p')a + \frac{i\epsilon}{2pk}\left(\int_{\xi}^{\xi+ka} - \int_{\xi'}^{\xi'+ka}\right) d\xi'' \right. \\ &\quad \left. \times(-2pA(\xi'') + \epsilon A^2(\xi''))\right] S^{-1}(L)\bar{E}(L(x+a)|p) \\ &\quad \times\langle 0|T\psi^{A'}(L(x+a))\bar{\psi}^{A'}(L(x'+a))|0\rangle E(L(x'+a)|p')\overline{S^{-1}}(L). \end{aligned} \tag{A.21}$$

Performing a gauge transformation

$$\psi^{A'}(L(x+a)) = \exp(-i\epsilon(\Lambda(L(x+a)) - \Lambda(x)))\psi^{A}(L(x+a))$$

in order to restore the original gauge dependence the first part of the integral, i.e. $\int(-2pA)$, cancels. This reflects the gauge invariance of the Volkov transform of the propagator. The second part, $\int \epsilon A^2$, cancels anyway since A^2 is constant.

We take now

$$a^{\mu} = -\tfrac{1}{2}(x+x')^{\mu} = -X^{\mu} \tag{A.22}$$

so that $L = L(X)$ and $\omega\hat{\epsilon} = kX$. Then equation (A.21) becomes

$$i\tilde{G}'(p,p') = \frac{1}{(2\pi)^4}\int d^4X \exp[i(p-p')X]S^{-1}(L(X))g(p,p')S(L(X)) \tag{A.23}$$

where

$$g(p,p') = \int d^4z \, \bar{E}(L(X)\tfrac{1}{2}z|p)\langle 0|T\psi^{A}(L(X)\tfrac{1}{2}z)\psi^{A}(-L(X)\tfrac{1}{2}z)|0\rangle E(-L(X)\tfrac{1}{2}z|p') \tag{A.24}$$

does not depend on X: due to gauge invariance it depends only on the scalar variables $pL(X)z$, $p'L(X)z$, $(L(X)z)^2$, $kL(X)z$ which are all independent of X. A complete basis of Dirac matrices is given by $\not{A}_i$ and $\not{B}_i\hat{\not{\epsilon}}_\pm$, where

$$\not{A}_i = 1, \not{k}, \not{p}, \tfrac{1}{2}[\not{\epsilon}_+, \not{\epsilon}_-], \gamma_5\not{k}, \gamma_5\not{p}, \not{p}\not{k}, \gamma_5$$

and

$$\not{B}_i = 1, \not{k}, \not{p}, \gamma_5$$

commute with $S(L)$. If we decompose $g(p,p')$ with respect to that basis

$$g(p,p') = \sigma_i(p,p')\not{A}_i + \sigma_i^{(+)}(p,p')\not{B}_i\hat{\not{\epsilon}}_+ + \sigma_i^{(-)}(p,p')\not{B}_i\hat{\not{\epsilon}}_-$$

and apply equation (A.18), we find

$$\sigma_i(p,p') \sim \delta(p-p'), \qquad \sigma_i^{(\pm)}(p,p') \sim \delta(p-p' \pm k)$$

as in equation (27). So this structure obtains to arbitrary order in α as asserted above.

References

Abramowitz M and Stegun I A 1965 *Handbook of Mathematical Functions* (New York: Dover)
Baier V N, Katkov V M, Milstein A I and Strachovenko V M 1975 *Zh. Eksp. Teor. Fiz.* **69** 783–99
Becker W 1976 *Proc. Conf. on Interaction of Coherent Electromagnetic Radiation with Matter, Technical University of Graz* (Graz: Technical University of Graz)
Becker W and Mitter H 1975 *J. Phys. A: Math. Gen.* **8** 1638–57
Bjørken J D and Drell S D 1965 *Relativistic Quantum Mechanics* (New York: McGraw-Hill)
Breitenlohner P and Mitter H 1968 *Nucl. Phys.* B **7** 443–58
Brown L S and Kibble T W B 1964 *Phys. Rev.* **133** A705–19
Goldman I I 1964 *Sov. Phys.-JETP* **19** 954–7
Mitter H 1975 *Acta Phys. Austr., Suppl. 14* 397–468
Nikishov A I and Ritus V I 1964 *Sov. Phys.-JETP* **19** 529–41
Oleinik V P 1967a *Sov. Phys.-JETP* **25** 697–708
—— 1967b *Sov. Phys.-JETP* **26** 1132–8
Reiss H R and Eberly J H 1966 *Phys. Rev.* **151** 1058–66
Richard J L 1972 *Nuovo Cim.* A **8** 485–500
Ritus V I 1967 *Sov. Phys.-JETP* **24** 19041–4
—— 1970 *Sov. Phys.-JETP* **30** 1181–7
—— 1972 *Ann. Phys., NY* **69** 555–82
Tsai W Y and Yildiz A 1973 *Phys. Rev.* D **8** 3446–60
Zeldovich Y B 1967 *Sov. Phys.-JETP* **24** 1006–8